

GM Math 7 Chapter 4 Notes – Mr. Adey – 2018

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Rational Numbers: Are numbers that can be represented by a fraction or ratio

Irrational Numbers: Are numbers that cannot be represented by a fraction or ratio.

Terminating decimal - A decimal that ends at a certain point

Repeating decimal – the decimal form of a fraction. Repeating decimals can be represented using bar notation. In bar notation, a bar is drawn only over the digit(s) that repeat

$$\frac{1}{11} = .\overline{09}$$

Non-Repeating & Non-Terminating Decimals – Never end and never repeat the same sequence of numbers.

Examples: $\sqrt{2}$ or π

They are always irrational numbers (because an irrational number cannot be represented by a fraction or ratio)

Writing fractions as decimals:

Method 1: Divide the numerator by the denominator

$$\frac{3}{4} \quad 3 \div 4 = 0.75$$

Method 2: If possible, convert the fraction to have a denominator of 100. The decimal is the numerator.

$$\frac{3}{25} = \frac{3}{25} \times \frac{4}{4} = \frac{12}{100} = 0.12$$

If the fraction is negative, simply follow method 1 or 2 and place a “-“ sign in front of the answer

Compare and Order Integers:

Use a number line or convert all the numbers into decimals or fractions to compare.

REMEMBER --- the larger a negative number, the farther it is from zero (and the smaller it actually is)

So -7 is less than -4

Example: Order from least to greatest: -7.78, $-7\frac{12}{100}$, $-7\frac{13}{20}$,

Convert all numbers into decimal form:

-7.78, -7.12, -7.65

OR

Convert all numbers into fraction form with same denominators:

$$-7\frac{78}{100}, -7\frac{12}{100}, -7\frac{65}{100}$$

Then put them into order. The smallest number is on the left.

$$-7\frac{78}{100} \text{ then } -7\frac{65}{100} \text{ and finally } -7\frac{12}{100}$$

Changing mixed number to improper fraction:

Example: $5\frac{2}{3}$

1) Multiply your **whole number** by the denominator. $5 \times 3 = 15$

2) ADD that answer to the numerator. $15 + 2 = 17$.

3) Put that number over the original denominator. $= \frac{17}{3}$

Equivalent Fractions:

Simply multiply the original fraction (both numerator and denominator) by the same number to get equivalent fractions:

Example:

$$\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$$

$$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

Estimating Fractions.

Use the benchmark number 0, $\frac{1}{2}$ or 1 to round each fraction in order to estimate.

$$\frac{7}{8} \rightarrow 1 \qquad \frac{1}{3} \rightarrow \frac{1}{2} \qquad \frac{1}{10} \rightarrow 0$$

Example:

$$4\frac{2}{9} + 6\frac{13}{27} \rightarrow 4 + 7 = 11$$

Steps for Adding / Subtracting Fractions

1) If denominators are not the same, make them the same using:

a) LCM **OR**

b) Equivalent fractions **OR**

c) Cross multiplying

2) Add or subtract numerator (top) fraction pieces

3) COPY the original denominator

4) Simplify if necessary

Example 1: (Like denominators)

$$\frac{5}{9} - \frac{2}{9} = \frac{3}{9}$$

Simplifies to $\frac{1}{3}$

Example 2: (Unlike denominators)

$$\frac{13}{16} - \frac{1}{4}$$

Change $\frac{1}{4} \rightarrow \frac{4}{16}$ (equivalent fraction)

$$\frac{13}{16} - \frac{4}{16} = \frac{9}{16}$$

Example 3: (Unlike denominators)

$$\frac{3}{4} + \frac{1}{3}$$

Use cross multiplying

$$3 \times 3 = 9 \qquad 4 \times 1 = 4 \qquad \text{denominator } 4 \times 3 = 12$$

$$9/12 + 4/12 = 13/12$$

Simplifies to $1 \frac{1}{12}$

Steps for Adding Fractions with Mixed numbers

1) If denominators are not the same, make them the same using:

a) LCM **OR**

b) Equivalent fractions **OR**

c) Cross multiplying

2) Add numerator (top) fraction pieces

3) COPY the original denominator

4) Add the whole numbers

5) Simplify if necessary, changing improper fractions to mixed numbers and adding the whole numbers

Example 1:

$$15 \frac{1}{6} + 7 \frac{1}{2}$$

Change $\frac{1}{2}$ to $\frac{3}{6}$ (Equivalent fractions)

$$15 \frac{1}{6} + 7 \frac{3}{6} = 22 \frac{4}{6}$$

$22 \frac{4}{6}$ simplifies to $22 \frac{2}{3}$

Example 2:

$$11 \frac{4}{5} + 5 \frac{7}{10}$$

Change $\frac{4}{5}$ to $\frac{8}{10}$ (Equivalent fractions)

$$11 \frac{8}{10} + 5 \frac{7}{10} = 16 \frac{15}{10}$$

$16 \frac{15}{10}$ simplifies to $16 + 1 \frac{5}{10}$ ($\frac{15}{10} = 1 \frac{5}{10}$)

$\frac{5}{10}$ simplifies to $\frac{1}{2}$, so the answer is $17 \frac{1}{2}$

Steps for Subtracting Fractions with Mixed numbers

1) If denominators are not the same, make them the same using:

a) LCM **OR**

b) Equivalent fractions **OR**

c) Cross multiplying

2) Subtract numerator (top) fraction pieces

IF the first fraction is smaller than the second, you must borrow from the whole number, then subtract

3) COPY the original denominator

4) Subtract the whole numbers

5) Simplify if necessary, changing improper fractions to mixed numbers and adding the whole numbers

Example 1:

$$12 \frac{5}{6} - 4 \frac{1}{2}$$

Change $\frac{1}{2}$ to $\frac{3}{6}$ (Equivalent fractions)

$$12 \frac{5}{6} - 4 \frac{3}{6} = 8 \frac{2}{6}$$

$8 \frac{2}{6}$ simplifies to $8 \frac{1}{3}$

Example 2:

$$15 \frac{3}{5} - 4 \frac{8}{10}$$

Change $\frac{3}{5}$ to $\frac{6}{10}$ (Equivalent fractions)

$$15 \frac{6}{10} - 4 \frac{8}{10}$$

Since you cannot take 8 away from 6, you must borrow $\frac{10}{10}$ from the 15

$$15 \frac{6}{10} \rightarrow 14 + 1 \frac{6}{10} \rightarrow 14 + \frac{10}{10} + \frac{6}{10} \rightarrow 14 \frac{16}{10}$$

$$14 \frac{16}{10} - 4 \frac{8}{10} = 10 \frac{8}{10}$$

Now simplify

$$= 10 \frac{4}{5}$$

Multiplying Fractions

With multiplying fractions, denominators do NOT have to be the same.

1. Multiply numerators straight across
2. Multiply denominators straight across
3. Simplify if necessary.

SPECIAL NOTES:

A. If you are multiplying a fraction by a whole number, turn the whole number into a fraction by putting it over 1 (put 1 as the denominator)

B. If you are taking a fraction of a number or a fraction of a fraction (e.g., $\frac{3}{5}$ of 5), change the word "OF" to multiply and follow Rule A

C. You can simplify BEFORE you multiply by checking for compatible numbers and reducing them diagonally. (See Examples 3 and 4)

Example 1:

$$\frac{3}{5} \text{ of } 10 \text{ IS } \frac{3}{5} \times \frac{10}{1} = \frac{30}{5} = 30 \div 5 = 6$$

Example 2:

$$\frac{4}{5} \times \frac{3}{4} = \frac{12}{20} = \frac{3}{5} \text{ (simplest form)}$$

You can SIMPLIFY before you multiply by dividing 4 and 4 by 4 (the GCF).

$$\frac{1}{5} \times \frac{3}{1} = \frac{3}{5}$$

Multiplying Mixed Numbers / Improper Fractions

When multiplying Mixed numbers, they MUST be in improper fraction form before you can multiply them.

Change mixed number to improper fractions

Then, follow the rules for multiplying fractions:

0. → Simplify / Reduce across fractions (if possible) before continuing

1. Multiply numerators across

2. Multiply denominators across

3. Simplify if necessary. NOTE: You may have to turn the improper fractions back to mixed numbers

Example:

$$4\frac{1}{5} \times 3\frac{8}{9}$$

$$\rightarrow 4 \times 5 = 20 + 1 = \frac{21}{5}$$

$$\rightarrow 3 \times 9 = 27 + 8 = \frac{35}{9}$$

So $4\frac{1}{5} \times 3\frac{8}{9}$ rewrites as:

$$\frac{21}{5} \times \frac{35}{9}$$

21 and 9 can be divided by 3, so this simplifies to:

$$\frac{7}{5} \times \frac{35}{3}$$

35 and 5 can be divided by 5, so this simplifies even more to:

$$\frac{7}{1} \times \frac{7}{3} = \frac{49}{3} = 16\frac{1}{3}$$

Obviously, 7×7 is a lot easier to work with than 21×35

Dividing Fractions

Since dividing is the inverse (opposite) operation from multiplying, when we multiply fractions we *invert* the **second** fraction and *then multiply the fractions as usual*.

When we invert (flip) the fraction, this is called the **reciprocal**.

AN EASY WAY TO REMEMBER THIS IS: LEAVE IT, CHANGE IT, FLIP IT

The reciprocal of $\frac{7}{3}$ is ...s ÷

$$\frac{7}{3} = \frac{3}{7}$$

Example:

$$\frac{1}{5} \div \frac{8}{9}$$

The reciprocal of $8/9$ is $9/8$

$$\frac{1}{5} \times \frac{9}{8} = \frac{9}{40}$$

9/40 cannot be simplified, so that is the final answer.

Dividing Fractions with Mixed Numbers

Follow the steps for changing mixed numbers to improper fractions.

Follow the steps for dividing fractions.

NOTE: DO NOT CROSS REDUCE OR SIMPLIFY BEFORE DIVIDING

Example:

$$3\frac{1}{5} \div 2\frac{3}{4}$$

$$\rightarrow 3 \times 5 = 15 + 1 = \frac{16}{5}$$

$$\rightarrow 2 \times 4 = 8 + 3 = \frac{11}{4}$$

$$\frac{16}{5} \div \frac{11}{4}$$

Find the reciprocal and **multiply**

$$\frac{16}{5} \times \frac{4}{11} \quad (\text{Cross simplify first if possible. Here, you can't})$$

$$= 1\frac{9}{55}$$

Changing Customary Units

Students should memorize the following units, if they don't already know it:

$$12 \text{ inches} = 1 \text{ ft}$$

$$3 \text{ ft} = 1 \text{ yd}$$

$$5,280 \text{ ft} = 1 \text{ mile}$$

$$8 \text{ oz} = 1 \text{ cup (liquid)}$$

$$2 \text{ c} = 1 \text{ pt}$$

$$2 \text{ pt} = 1 \text{ qt}$$

$$4 \text{ qt} = 1 \text{ gal}$$

$$16 \text{ oz} = 1 \text{ lb (solid)}$$

$$2,000 \text{ lb} = 1 \text{ T}$$

To convert between customary and metric units, multiply by fractions.

CONVERSION FACTORS

LENGTH-DISTANCE

Inches (in.)	x 25.4	= Millimeters (mm)	x .0394	= Inches
Feet (ft.)	x .305	= Meters (m)	x 3.281	= Feet
Miles	x 1.609	= Kilometers (km)	x .0621	= Miles

VOLUME

Cubic Inches (in ³)	x 16.387	= Cubic Centimeters	x .061	= in ³
IMP Pints (IMP pt.)	x .568	= Liters (L)	x 1.76	= IMP pt.
IMP Quarts (IMP qt.)	x 1.137	= Liters (L)	x .88	= IMP qt.
IMP Gallons (IMP gal.)	x 4.546	= Liters (L)	x .22	= IMP gal.
IMP Quarts (IMP qt.)	x 1.201	= US Quarts (US qt.)	x .833	= IMP qt.
IMP Gallons (IMP gal.)	x 1.201	= US Gallons (US gal.)	x .833	= IMP gal.
Fl. Ounces	x 29.573	= Milliliters	x .034	= Ounces
US Pints (US pt.)	x .473	= Liters (L)	x 2.113	= Pints
US Quarts (US qt.)	x .946	= Liters (L)	x 1.057	= Quarts
US Gallons (US gal.)	x 3.785	= Liters (L)	x .264	= Gallons

MASS-WEIGHT

Ounces (oz.)	x 28.35	= Grams (g)	x .035	= Ounces
Pounds (lb.)	x .454	= Kilograms (kg)	x 2.205	= Pounds

PRESSURE

Pounds Per Sq. In. (psi)	x 6.895	= Kilopascals (kPa)	x .145	= psi
Inches of Mercury (Hg)	x .4912	= psi	x 2.036	= Hg
Inches of Mercury (Hg)	x 3.377	= Kilopascals (kPa)	x .2961	= Hg
Inches of Water (H ₂ O)	x .07355	= Inches of Mercury	x 13.783	= H ₂ O
Inches of Water (H ₂ O)	x .03613	= psi	x 27.684	= H ₂ O
Inches of Water (H ₂ O)	x .248	= Kilopascals (kPa)	x 4.026	= H ₂ O

TORQUE

Pounds-Force Inches (in-lb)	x .113	= Newton Meters (N·m)	x 8.85	= in-lb
Pounds-Force Feet (ft-lb)	x 1.356	= Newton Meters (N·m)	x .738	= ft-lb

VELOCITY

Miles Per Hour (MPH)	x 1.609	= Kilometers Per Hour (KPH)	x .621	= MPH
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POWER

Horsepower (Hp)	x .745	= Kilowatts	x 1.34	= Horsepower
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FUEL CONSUMPTION*

Miles Per Gallon IMP (MPG)	x .354	= Kilometers Per Liter (Km/L)
Kilometers Per Liter (Km/L)	x 2.352	= IMP MPG
Miles Per Gallon US (MPG)	x .425	= Kilometers Per Liter (Km/L)
Kilometers Per Liter (Km/L)	x 2.352	= US MPG

*It is common to convert from miles per gallon (mpg) to liters/100 kilometers (l/100 km), where mpg (IMP) x 1/100 km = 282 and mpg (US) x 1/100 km = 235.

TEMPERATURE

Degree Fahrenheit (°F)	= (°C x 1.8) + 32
Degree Celsius (°C)	= (°F - 32) x .56